Introduction of SEU International Summer School Program

Theme

2021 International Summer Courses on Mathematics and Statistics, Southeast University

Overview

This program will select appropriate problem models from the cutting-edge aspects of mathematics and statistics respectively, and introduce the latest research results in the related fields, in order to improve the understanding and utilization of knowledge for students. The emphasis of both theory and application is the highlight of this course. In addition, the reflection of the interdisciplinary cross-integration is also the main goal of this course. The program consists of three 24-period short online courses with 1 credit for each course.

Schedule (Draft)

July 5, 2021 — August 1, 2021

1 Mini Course：Selected Topics in Modern Mathematics

- **Hours/Credits**: 24 hours/ 1 credit

- **Lecturer:**

We will invite the teaching team from University of Luxembourg to teach this course, including:

Geometry： Prof. Dr. Jean-Marc Schlenker;

Analysis： Dr. Fei Pu;

Algebra： Prof. Dr. Antonella Perucca (course 1) and Prof. Dr. Gabor Wiese (course 2);

Probability and Statistics： Prof. Dr. Ivan Nourdin (course 1) and Prof. Dr. Mark Podolskij (course 2).

- **Description**

======== GEOMETRY ========

Teacher: Prof. Dr. Jean-Marc Schlenker

Title: The geometry of polyhedra in Euclidean space

Abstract: We intend to present some classical and more recent results on the geometry of convex polyhedra in Euclidean space, as well as some open problems of current interest. The course could fit over 2 sessions of 135mn. Assessment could be done through a few multiple-choice of numerical-answer questions in a moodle-type test.

======== ANALYSIS ========

Teacher: Dr. Fei Pu

Title: Basics of Fourier Analysis

Abstract: I am planning to follow Stein's book to present some basic materials on Fourier Analysis and the key words are: Fourier inversion, Plancherel identity, Poisson summation formula, Theta and zeta functions.

======== ALGEBRA ========

Teachers: Prof. Dr. Antonella Perucca (course 1) and Prof. Dr. Gabor Wiese (course 2)

Title: Finite fields: from the cyclicity of the unit group to Artin's conjecture on primitive roots, Gauss' quadratic reciprocity law, primality tests and the Langlands program

Abstract:
Part 1 - Artin’s Conjecture for primitive roots and related problems (A. Perucca)
We start by considering the unit group (Z/pZ)* of the integers modulo a prime number p, and then investigate the multiplicative order and index of an element in this group. By varying the prime number, distribution questions naturally arise, the most famous being the one addressed in Artin’s Conjecture for primitive roots. To understand the conjecture and its heuristics we introduce cyclotomic number fields and Kummer extensions. To conclude we present recent results on this topic obtained by mathematicians in Luxembourg.

Part 2 - A primality test, quadratic reciprocity, and more general reciprocity laws (G. Wiese)
From Part 1, we know that half of the elements in (Z/pZ)* are squares and half are non-squares. The famous quadratic reciprocity law conjectured by Euler and proved by Gauss relates this for two primes: say p_1,p_2 are two primes that are 1 mod 4; then p_1 is a square mod p_2 if and only if p_2 is a square mod p_1. This law can be proved using cyclotomic fields, introduced in Part 1. As a practical application of quadratic reciprocity, we introduce the Solovay-Strassen primality test for deciding if a given positive integer is a prime number or not.
In a final part, we give some hints on generalisations of quadratic reciprocity leading us (vaguely) to the Langlands program.

======== **PROBABILITY AND STATISTICS** ========

Teachers: Prof. Dr. Ivan Nourdin (course 1) and Prof. Dr. Mark Podolskij (course 2)

Title: Large random matrices

Abstract:
Part 1: “Free probability and large random matrices”
- Reminder of the classical central limit theorem
- Random matrices
- Concept of classical and free independence
- Stieltjes transform
- Semicircular and Marcenko-Pastur laws
- Voiculescu's theorem, and an alternative proof of Wigner's theorem
- Classic and free Brownian motion

Part 2: “Estimation of large covariance matrices”
-Empirical covariance matrices
-Principal component analysis
-Asymptotic theory for empirical eigenvalues
-Estimation of high-dimensional covariance matrices
-Relations to random matrix theory

➢ Tentative schedule

Day	Start time	End time	Teacher	Topic
July 5 (Monday)	8:45am (UL time) 2:45pm (SEU time)	11am (UL time) 5pm (SEU time)	Prof. Dr. Jean-Marc Schlenker	Geometry
July 8 (Thursday)	8:45am (UL time) 2:45pm (SEU time)	11am (UL time) 5pm (SEU time)	Prof. Dr. Jean-Marc Schlenker	Geometry
July 12 (Monday)	8:45am (UL time) 2:45pm (SEU time)	11am (UL time) 5pm (SEU time)	Prof. Dr. Antonella Perucca	Algebra
July 15 (Thursday)	8:45am (UL time) 2:45pm (SEU time)	11am (UL time) 5pm (SEU time)	Prof. Dr. Gabor Wiese	Algebra
July 19 (Monday)	8:45am (UL time) 2:45pm (SEU time)	11am (UL time) 5pm (SEU time)	Dr Fei Pu	Analysis

July 22 (Thursday)	8:45am (UL time) 2:45pm (SEU time)	11am (UL time) 5pm (SEU time)	Dr Fei Pu	Analysis
July 26 (Monday)	8:45am (UL time) 2:45pm (SEU time)	11am (UL time) 5pm (SEU time)	Prof. Dr. Ivan Nourdin	Probability and Statistics
July 29 (Thursday)	8:45am (UL time) 2:45pm (SEU time)	11am (UL time) 5pm (SEU time)	Prof. Dr. Mark Podolskij	Probability and Statistics

2 Mini Course: Selected Topics in Frontier of Scientific Computation

2.1 Mini Course: Selected Topics in Frontier of Scientific Computation (Part I) Topic: Machine Learning and Design optimization under uncertainty

➢ **Hours/Credits:** 12 hours/ 0.5 credit

➢ **Lecturer:**

Matin Stynes
Beijing Computational Science Research Center
(m.stynes@csrc.ac.cn，http://www.csrc.ac.cn/en/people/faculty/151.html）

➢ **Description**

In convection-diffusion problems, when the elliptic operators are multiplied by some parameter that is allowed to be close to zero, the first-order convective derivatives no longer play a relatively minor role is the system. Instead, it also has a strong influence on the solution of the boundary value problem. The convection-diffusion problems have been widely used in application, such as the Black-Scholes equation in finance, the linearized Navier-Stokes equation with large Reynolds number. The goal of this course is to introduce the background of the one-dimensional convection-diffusion equations and the basic theory of analytical solution and the finite difference method.

➢ **Prerequisites**

Calculus, Linear Algebra, Differential Equations, Numerical Analysis. Students are strongly encouraged to use MATLAB for programming.

➢ **Textbooks**

Martin Stynes, David Stynes, Convection-Diffusion Problems：An Introduction to Their Analysis and Numerical Solution, The American Mathematical Society.

- **Course objectives**

After this course, students should be able to

- Understand the background of the convection-diffusion problems
- Understand the fundamental theory of the one-dimensional convection- diffusion problems
- Master the finite difference method for the one-dimensional convection- diffusion problems and its convergence analysis

- **Class schedule**

Hours 1-2	Introduction to the convection-diffusion problems by some motivating examples
Hours 3-4	Maximum principle and asymptotic expansion
Hours 5-6	Asymptotic analysis to the convection-diffusion problems, Green's formula
Hours 7-8	A priori bounds on the solution and decompositions of the solution
Hours 9-10	Upwinding scheme for solving the convection-diffusion problems
Hours 11-12	Shishkin meshes, uniformly convergent schemes

- **Evaluation methods:**

Project.

2.2 Mini Course: Selected Topics in Frontier of Scientific Computation (Part II)

Topic: Introduction to Numerical Methods for Stochastic Differential Equations

- **Hours/Credits:** 12 hours/ 0.5 credit

- **Lecturer:**

Yanzhao Cao
Department of Mathematics & Statistics,
Auburn University
(yzc0009@auburn.edu, http://webhome.auburn.edu/~yzc0009/)

- **Description**

In this short course, I will introduce numerical methods for stochastic differential equations, which have been used widely used in biology, finance and engineering. Topics include Brownian motion and stochastic calculus in linear and nonlinear equations, analytic and numerical methods for SDEs, and parameter estimation for SDEs.

- **Prerequisites**

Calculus, linear algebra, differential equations and probability. Students are strongly encouraged to use MATLAB for programming.

- **Textbooks**

There will be no textbooks but lecture notes will be provided.

- **Course objectives**

After this course, students should be able to

- Learn the background and application to the mathematical models with random parameters or stochastic disturbance
- Master basic algorithms for solving problems with stochastic disturbance or random parameters
- Learn the algorithms to stochastic computation based on machine learning

- **Class schedule**

Hours 1-2	Introduction to stochastic differential equations, including some motivating examples.
Hours 3-4	Random walk, Brownian motion and stochastic calculus, and stochastic differential equations
Hours 5-6	Strong solutions, Well-posedness, Solution techniques
Hours 7-8	Basic concepts of numerical methods for stochastic differential equations, simulation of white and color noises Numerical methods for linear equations: stability and convergence
Hours 9-10	Numerical methods for nonlinear equations. Stiffness and treatment
Hours 11-12	Parameter estimation or stochastic differential equations

- **Evaluation methods:**

Project.

3 Mini Course: Categorical Data Analysis

- **Hours/Credits:** 24 hours/ 1 credit

- **Lecturer:**

Prof. Weixin Yao
Department of Statistics, University of California, Riverside

- **Description:**

The content mainly includes semi-parametric and non-parametric statistics, robust statistical models, high-latitude data and statistical analysis of big data, etc. Semi-parametric and non-parametric statistical models have a wide range of applications, and their assumptions are weaker than traditional parametric models, so they are more widely used especially in the era of big data, when statistical inferences tend to be more accurate. The data collected today often have outliers, and traditional statistical inferences such as the least square method for these outliers are very unstable and often lead to false inferences. Robust statistical models are not affected by these outliers and can provide robust and reliable statistical inferences. In the era of big data, a lot of data is high latitude. Traditional statistical analysis methods are often not applicable at this time. This course will introduce a series of high latitude statistical methods and some big data statistical calculation methods.

- **Prerequisites:**

Calculus, Linear Algebra, Differential Equations, Real Analysis, Complex Analysis, Probability Theory, Mathematical Statistics, Random Processes

- **Textbooks:** Notes

- **Schedule:** July 5, 2021 — August 1, 2021

- **Evaluation:** Project or Paper Test

Number of Participants

200

Application Requirements

It is mainly for the undergraduate students who have finished the second year courses in mathematics or statistics

Application Deadline

June 15, 2021

Host & Organizer

Southeast University / School of Mathematics

Contact Information

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