



## **2021 International Summer Courses on Mathematics and Statistics**



### **Summary Report**

China·Nanjing

August, 2021

# International Summer School Program at Southeast University

## 2021 International Summer Courses on Mathematics and Statistics



### Overview

This program will select appropriate problem models from the cutting-edge aspects of mathematics and statistics respectively, and introduce the latest research results in the related fields, in order to improve the understanding and utilization of knowledge for students. The emphasis of both theory and application is the highlight of this course. In addition, the reflection of the interdisciplinary cross-integration is also the main goal of this course. The program consists of three 24-hour short online courses with 1 credit for each course.

### Course 1: Selected Topics in Modern Mathematics

Hours/Credits: 24 hours/ 1 credit

Lecturer: We will invite the team from University of Luxembourg to teach this course, including: Prof. Dr. Jean-Marc Schlenker, Dr. Fei Pu, Prof. Dr. Antonella Perucca, Prof. Dr. Gabor Wiese, Prof. Dr. Ivan Nourdin and Prof. Dr. Mark Podolskij.

### Course 2: Selected Topics in Frontier of Scientific Computation

#### Part I: Machine Learning and Design optimization under uncertainty

Hours/Credits: 12 hours/ 0.5 credit

Lecturer: Prof. Matin Stynes, Beijing Computational Science Research Center

#### Part II: Introduction to Numerical Methods for Stochastic Differential Equations

Hours/Credits: 12 hours/ 0.5 credit

Lecturer: Prof. Yanzhao Cao, Department of Mathematics & Statistics, Auburn University

### Course 3: Categorical Data Analysis

Hours/Credits: 24 hours/ 1 credit

Lecturer: Prof. Weixin Yao, Department of Statistics, University of California, Riverside

### Time Period

Jul. 5 – Aug. 1, 2021

### Requirements

Mainly for the students who have finished 2nd year undergraduate courses in math or statistics

### Number of Participants

200

### Application Deadline

Jun. 15, 2021

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**COURSE 1: SELECTED TOPICS IN MODERN MATHEMATICS**

**Hours/Credits** 24 hours(July 5 - August 1, 2021 ) / 1 credit  
Wednesday and Friday 14:45 pm - 17:00 pm  
Platform: Zoom + QQ

**Description** "Selected Topics in Modern Mathematics" is a compulsory course for all majors of mathematics. The course content will be selected by the lecturer according to his professional expertise and in combination with the development of modern mathematics, including classical hot topics in analysis, algebra, geometry and other branches. Focusing on several mathematical problems, the lecturer will introduce the research history, study methods and latest advances in detail, to broaden the students' academic vision and enhance the students' interest in mathematics.

**Instructor** A teaching team from University of Luxembourg teach this course, including:



Prof. Dr. Jean-Marc Schlenker (**Geometry**)



Dr. Fei Pu (**Analysis**)



Prof. Dr. Antonella Perucca (course 1) (**Algebra**)



Prof. Dr. Gabor Wiese (course 2) (**Algebra**)



Prof. Dr. Ivan Nourdin (course 1) (**Probability and Statistics**)



Prof. Dr. Mark Podolskij (course 2) (**Probability and Statistics**)

## PREREQUISITES

Calculus, Linear algebra. It will be helpful if students have preliminary knowledge of Real analysis, Complex analysis, Abstract algebra, Probability and Statistics.

## COURSE OBJECTIVES

### ===== GEOMETRY =====

**Title:** The geometry of polyhedra in Euclidean space

**Abstract:** We intend to present some classical and more recent results on the geometry of convex polyhedra in Euclidean space, as well as some open problems of current interest. The course could fit over 2 sessions of 135mn. Assessment could be done through a few multiple-choice of numerical-answer questions in a moodle-type test.

### ===== ANALYSIS =====

**Title:** Basics of Fourier Analysis

**Abstract:** I am planning to follow Stein's book to present some basic materials on Fourier Analysis and the key words are: Fourier inversion, Plancherel identity, Poisson summation formula, Theta and zeta functions.

### ===== ALGEBRA =====

**Title:** Finite fields: from the cyclicity of the unit group to Artin's conjecture on primitive roots, Gauss' quadratic reciprocity law, primality tests and the Langlands program

**Abstract:**

**Part 1** - Artin's Conjecture for primitive roots and related problems (A. Perucca)  
We start by considering the unit group  $(\mathbb{Z}/p\mathbb{Z})^*$  of the integers modulo a prime number  $p$ , and then investigate the multiplicative order and index of an element in this group. By varying the prime number, distribution questions naturally arise, the most famous being the one addressed in Artin's Conjecture for primitive roots. To understand the conjecture and its heuristics we introduce cyclotomic number fields and Kummer extensions. To conclude we present recent results on this topic obtained by mathematicians in Luxembourg.

**Part 2** - A primality test, quadratic reciprocity, and more general reciprocity laws (G. Wiese)

From Part 1, we know that half of the elements in  $(\mathbb{Z}/p\mathbb{Z})^*$  are squares and half are non-squares. The famous quadratic reciprocity law conjectured by Euler and proved

by Gauss relates this for two primes: say  $p_1, p_2$  are two primes that are  $1 \pmod{4}$ ; then  $p_1$  is a square mod  $p_2$  if and only if  $p_2$  is a square mod  $p_1$ . This law can be proved using cyclotomic fields, introduced in Part 1. As a practical application of quadratic reciprocity, we introduce the Solovay-Strassen primality test for deciding if a given positive integer is a prime number or not.

In a final part, we give some hints on generalisations of quadratic reciprocity leading us (vaguely) to the Langlands program.

===== **PROBABILITY AND STATISTICS** =====

**Title:** Large random matrices

**Abstract:**

**Part 1-** “Free probability and large random matrices”

- Reminder of the classical central limit theorem
- Random matrices
- Concept of classical and free independence
- Stieltjes transform
- Semicircular and Marcenko-Pastur laws
- Voiculescu's theorem, and an alternative proof of Wigner's theorem
- Classic and free Brownian motion

**Part 2-** “Estimation of large covariance matrices”

- Empirical covariance matrices
- Principal component analysis
- Asymptotic theory for empirical eigenvalues
- Estimation of high-dimensional covariance matrices
- Relations to random matrix theory

## CLASS SCHEDULE

| Day                | Teacher                       | Topic    |
|--------------------|-------------------------------|----------|
| July 7 (Wednesday) | Prof. Dr. Jean-Marc Schlenker | Geometry |
| July 9 (Friday)    | Prof. Dr. Jean-Marc Schlenker | Geometry |

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|                     |                             |                               |
|---------------------|-----------------------------|-------------------------------|
| July 14 (Wednesday) | Prof. Dr. Antonella Perucca | Algebra                       |
| July 16 (Friday)    | Prof. Dr. Gabor Wiese       | Algebra                       |
| July 21 (Wednesday) | Dr Fei Pu                   | Analysis                      |
| July 23 (Friday)    | Dr Fei Pu                   | Analysis                      |
| July 28 (Wednesday) | Prof. Dr. Ivan Nourdin      | Probability and<br>Statistics |
| July 30 (Friday)    | Prof. Dr. Mark Podolskij    | Probability and<br>Statistics |

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## FEEDBACK FROM STUDENTS

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### COMMENT 1

虽然因为是英语教学让我在上课听讲时碰到了许多困难,但是通过对老师发的课件对课程进行预习和复习我还是跟上了老师的进度。老师讲授清楚,重点突出,层次分明,逻辑清晰,能耐心认真地对待我们课内外提出的问题。我觉得《现代数学选讲》这门课程有效地增进了我的知识和能力。

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### COMMENT 2

这门课程内容是由每一位主讲教师根据自身专业特长,结合现代数学的发展,选择了分析、代数、几何等分支中的经典热门话题,围绕几个数学问题来为我们详细介绍这个主题的研究历史、处理手法以及最新进展。我觉得很好的开阔了我们的学术视野,也提升我们对于数学的学习兴趣,帮助我们探寻数学蕴含的美丽。

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### COMMENT 3

现代数学选讲这门课程选取了分析、几何、代数、统计等多方面的知识,分别由不同教授来为我们讲解。其中,欧几里得空间中多面体的几何、傅里叶分析基础这两门课程让我印象深刻,普朗切雷尔恒等式、泊松求和公式、Theta 和 zeta 函数都会后面的学习中用到。美中不足的是,全英文教学对大部分同学来说还是有点困难,对于我而言,主要难的是数学名词和数学专业性句子的翻译。如果能提供课程的录像,相信会能够有更好的学习效果。



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#### COMMENT 4

在参与了暑期学校《现代数学选讲》这门课程后，我对现代前沿数学有了更为深刻的认识，首先，该门课程分为四个部分：1. Polyhedral geometry（多面体几何）2. Algebra（代数）3. Analysis（分析）4. Probability and Statistics（概率与统计）。其中最让我收获颇多的课程便是多面体几何以及分析课程。Dr. Jean-Marc Schlenker 老师为我们讲解了多面体几何的一些前沿理论，结合实际与建筑化抽象为具体，让我们更清晰地理解了多面体几何的构成和一些有用的结论。

另外，对于分析课程，我们深入学习了傅里叶变换的原理以及其应用，这是现代数学研究中重要的一环，对其的深入探讨，可以加深我们对分析学的理解，为以后对数学的研究打下基础。总之，这四门课都对我们未来数学的研究打下了很好的基础，外教授课更是锻炼了我们接收国外先进理论的能力，很荣幸能有这次暑期学校的学习机会。

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#### COMMENT 5

四周的暑期学校课程，我们简单了解了几何、代数、分析和概率统计四个数学方向更加深层次的知识体系，我学会了很多新的知识和解决问题的方法，尤其是代数方向中数论、群论、环论和域论的结合让我觉得非常有趣。当然，一开始我在听课上的问题还是很大的，不过在老师减慢讲课速度之后就逐渐适应了授课强度。对于这门课，我希望课程具有更多的科普性质，继续往更多的方向扩散，或者专心于一个方向，深入讨论这个方向的知识。另外，因为是全英文授课，如果每一位老师都有自己较为成熟的笔记，并且控制讲课的速度的话，课堂的授课效果会更好。

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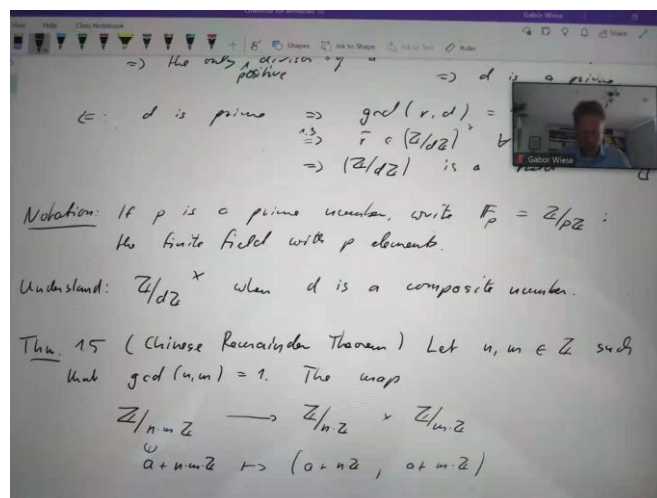
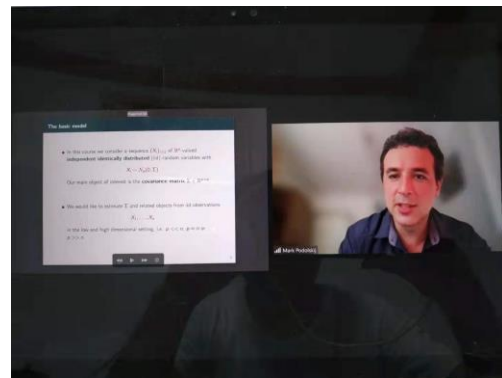
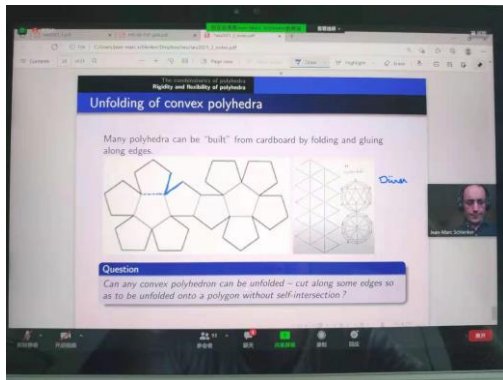
#### COMMENT 6

在暑期学校里，我们修了现代数学选讲这门课程。课程的开始阶段还并没有那么困难，随着课程的深入，越来越多我们不熟悉的专业术语出现，课程难度也逐步提升，对我的英语听力及阅读能力也是不小的挑战，幸运的是，我们的老师十分关心我们的学习情况，给我们对课程内容进行了一些预习内容的补充，帮助我们克服困难。

课程涉及几何，数论，分析和统计，扩充了我们的知识面，也在我们的后续很多课程中有相应的应用。全英文授课的方式为我们以后阅读英文文献和听英文报告打下了一定的基础，但是如果有条件提供一些字幕，不管是英文还是中文，都会对同学们的学习有很大的帮助。又或者如果是线下上课，也可以相应推荐一些中文参考书帮助同学们过渡。

## FEEDBACK FROM TEACHERS

《现代数学选讲》是在数学专业本科生二年级结束时开设的一门课程。课程采用了 ZOOM 和 QQ 相结合的线上教学方式。主讲人通过选取一些难度适中的主题，介绍分析、代数、几何、概率这些基础课的理论应用和发展前景，一方面对之前所学知识进行融会贯通，另一方面为后续课程学习做好铺垫。总体而言，课程基本上达到了预期的目标。同学们一致表示通过此门课程的学习感到受益匪浅，但在英语帮扶、师生互动方面还有待加强。



## Selected Topics in Modern Mathematics ↗

聊天 公告 相册 文件 作业 设置 ▾

我觉得是因为rapidly decreasing

07219120 石珂汗(1251963742) 2021/7/21 17:30:24

· 欧拉公式? 复数模小于等于1

07119119 邓翔&lt;thegreatdx@qq.com&gt; 2021/7/21 17:31:20

 $(\mathbb{R})$ A measurable function  $f : \mathbb{R} \mapsto \mathbb{C}$  is called **rapidly decreasing** if

$$\sup_{x \in \mathbb{R}} |x|^k |f^{(\ell)}(x)| < \infty, \quad \text{for every } k, \ell \geq 0.$$

王小六(1270830107) 2021/7/21 17:32:27

是的

王小六(1270830107) 2021/7/21 17:32:35

- 请先理解这个定义
- 然后知道f 在正负无穷处都是趋于0的
- $e^{\dots}$  这个是有界的
- @07219120 石珂汗

07219120 石珂汗(1251963742) 2021/7/21 17:33:50

· 明白了

## Selected Topics in Modern Mathematics ↗

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我布置的作业

2021-7-29 星期四



数学作业

Homework 4



数学作业

Homework 3



数学作业

Homework 1

2021-7-16 星期五



数学作业

Homework 2.

## COURSE 2: SELECTED TOPICS IN FRONTIER OF SCIENTIFIC COMPUTATION

### Part I: Introduction to Numerical Methods for One-dimensional Convection-diffusion Equation

**Hours/Credits** 12 hours (July 5 — July 18, 2021)/ 0.5 credit

Monday and Tuesday 8:00 am – 10:35 am

Platform: Tencent Meeting + QQ

#### Description

In convection-diffusion problems, when the elliptic operators are multiplied by some parameter that is allowed to be close to zero, the first-order convective derivatives no longer play a relatively minor role in the system. Instead, it also has a strong influence on the solution of the boundary value problem. The convection-diffusion problems have been widely used in application, such as the Black-Scholes equation in finance, the linearized Navier-Stokes equation with large Reynolds number. The goal of this course is to introduce the background of the one-dimensional convection-diffusion equations and the basic theory of analytical solution and the finite difference method.

#### Instructor

Matin Stynes (Beijing Computational Science Research Center)

Homepage: <http://www.csrc.ac.cn/en/people/faculty/151.html>



主讲人: Prof. Matin  
Stynes

北京计算科学研究中心国家“千人计划”专家，奇异摄动微分方程数值方法领域国际领军学者，曾任工业和应用数学学会（SIAM）英国和爱尔兰分会主席（2003-2005），曾任 SIAM Journal on Numerical Analysis 编委，目前担任 Advances in Computational Mathematics, Computational Methods in Applied Mathematics, Mathematical Proceedings of the Royal Irish Academy 等学术期刊编委。

## PREREQUISITES

Calculus, Linear Algebra, Differential Equations, Numerical Analysis. Students are strongly encouraged to use MATLAB for programming.

### COURSE OBJECTIVES

After this course, students should be able to

- Understand the background of the convection-diffusion problems
- Understand the fundamental theory of the one-dimensional convection- diffusion problems
- Master the finite difference method for the one-dimensional convection- diffusion problems and its convergence analysis

### CLASS SCHEDULE

|             |   |
|-------------|---|
| Hours 1-2   | Introduction to the convection-diffusion problems by some motivating examples |
| Hours 3-4   | Maximum principle and asymptotic expansion                                    |
| Hours 5-6   | Asymptotic analysis to the convection-diffusion problems, Green's formula     |
| Hours 7-8   | A priori bounds on the solution and decompositions of the solution            |
| Hours 9-10  | Upwinding scheme for solving the convection-diffusion problems                |
| Hours 11-12 | Shishkin meshes, uniformly convergent schemes                                 |
| Hours 13-15 | Comments on Homework, numerical experiment training, course summary           |

### Part II: Introduction to Numerical Methods for Stochastic Differential Equations

**Hours/Credits**                      12 hours (July 19 — August 1, 2021)/ 0.5 credit  
Monday and Tuesday 8:00 am – 10:35 am  
Platform:    Tencent Meeting + QQ

**Description**                              In this short course, I will introduce numerical methods for stochastic differential equations, which have been used widely used in biology, finance and engineering. Topics include Brownian motion and stochastic calculus in linear and nonlinear equations, analytic and numerical methods for SDEs, and parameter estimation for SDEs.

**Instructor**

Yanzhao Cao (Department of Mathematics &amp; Statistics)

Homepage: <http://webhome.auburn.edu/~yzc0009/>

主讲人：曹延昭 教授

美国奥本大学数学与统计系教授，国家天元数学东北中心执行委员会主任。1983年毕业于吉林大学数学系，1996年获弗吉尼亚理工学院数学博士学位，主要从事偏微分方程和积分方程数值解法、随机偏微分方程数值解、非线性滤波、不确定性量化等领域的研究，部分重要研究成果发表在《SIAM J. Numer. Anal.》、《Numer. Math.》、《Math. Comp.》、《IMA J. Numer. Anal.》等计算数学国际顶尖杂志。现担任包括计算数学国际顶尖期刊《SIAM J. Numer. Anal.》在内的多个学术期刊编委。

**PREREQUISITES**

Calculus, linear algebra, differential equations and probability. Students are strongly encouraged to use MATLAB for programming.

**COURSE OBJECTIVES**

After this course, students should be able to

- Learn the background and application to the mathematical models with random parameters or stochastic disturbance
- Master basic algorithms for solving problems with stochastic disturbance or random parameters
- Learn the algorithms to stochastic computation based on machine learning

**CLASS SCHEDULE**

|           |   |
|-----------|---|
| Hours 1-2 | Introduction to stochastic differential equations, including some motivating examples.                            |
| Hours 3-4 | Random walk, Brownian motion and stochastic calculus, and stochastic differential equations                       |
| Hours 5-6 | Strong solutions and its well-posedness   |
| Hours 7-8 | Basic concepts of numerical methods for stochastic differential equations, simulation of white and color noises ; |

|             |  |
|-------------|--|
|             | Numerical methods for linear equations: stability and convergence  |
| Hours 9-10  | Numerical methods for nonlinear equations: Stiffness and treatment |
| Hours 11-12 | Parameter estimation or stochastic differential equations          |
| Hours 13-14 | Numerical experiment training-Euler-type methods                   |
| Hours 15-16 | Numerical experiment training-Milstein-type methods                |
| Hours 17-18 | Comments on Homework, course summary                               |
| Hours 19-20 | Q&A  |

## FEEDBACK FROM STUDENTS

### COMMENT 1

在短短的一个月的暑期学校里，所有数学学院的学生都学习了两门科学计算相关的课程。第一个课程通过对流扩散方程引入了多种可行的计算微分方程的方法，开阔了我们的眼界，特别是全英文授课的方式，让我们熟悉了很多数学名词的英文表达，锻炼了我们的学术交流能力。第二个课程为我们介绍了随机微分方程，让我们了解了数学中的一个未曾了解过的分支。特别是对我们计算科学的学生来说，随机微分方程也为我们后续学习金融模型计算奠定了一定基础。总之在暑期学校开设的这门课程里，不少同学都收获了很多。

当然，这门课程还有一些可以提高的空间。首先是对于英文基础不好的同学，全英文授课的难度还是有些难以克服，我想可以通过课程录屏并且自动加字幕的方式，让没能听懂的同学后续可以看录屏跟上。其次就是可能部分学生存在对课程不感兴趣的现象，我认为这也很正常，因为科学计算的内容主要是涉猎到了计算科学同学的专业相关，对于统计和应数的学生来说，可能课题本身和专业重合度不大，如果明年的暑期学校，存在一些选课的自由度，可能同学们的学习效果会好一些。

### COMMENT 2

暑期的科学计算前沿选讲课程由几位不同的老师上课这一点比较好，可以接触到不同的研究内容和不同的老师风格。全英文授课对我们是好事，不过在课程之初有些困难，难以理解授课内容，难以准确看懂讲义或者 PPT，适应一段时间后有所改善，能听懂的内容会变多。在大家适应全英文的艰难磨合期，曹老师线上进行课后讲解对我们有很大帮助，相比之下还是中文更容易接受。线上教学的助教工作也很到位，分组安排助教提交作业，且按照每周作业构成成绩考核的方式比较合适，把学习任务分担在平时。总体上课程非常有序并且我们很有收获。

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### COMMENT 3

该课程 4 周 8 次，比较紧凑，由 Martin 和曹延昭两位老师主讲，以线上的形式展开。Martin 教授主要讲授了一维对流扩散方程的数值算法，由于英文授课，老师依据学生们的接受程度调整讲课速度，替学生考虑。并在课尾留有时间让学生提问题，耐心细致的答疑。在课后留有适度的作业且有多个助教跟进，学生们有疑惑也可以在 QQ 群进行讨论。曹延昭老师主要讲授了随机微分方程及数值近似，在课上用手写板的形式讲授，给学生们一种在线下课堂上课的体验，并且从所需基础知识讲起，一步步引导学生，让学生们更易接受较难内容的学习。在 Martin 老师全英文讲课过程中，曹婉容老师另外安排时间帮助我们梳理了重难点，对于课程内容的理解帮助很大。另外，课程的助教非常认真负责，大概一共有 6 名助教，分组收作业，反馈及时，效率很高。通过这次暑期学习，我了解到了科学计算的其他前沿领域，收获很多，同时也意识到课后还需花更多的时间去学习。

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### COMMENT 4

在暑期学校里，我们修了科学计算前沿选讲这门课程。课程的开始阶段还并没有那么困难，随着课程的深入，越来越多我们不熟悉的专业术语出现，课程难度也逐步提升，对我的英语听力及阅读能力也是不小的挑战，当然幸运的是，我们的曹老师十分关心我们的学习情况，另找时间给我们对课程内容进行总结，帮助我们克服困难。

第一个课程介绍了对流扩散方程及相应的一些性质，并给出了一些微分方程的求解。第二个课程则是为我们介绍了随机微分方程，它在我们的后续很多课程中也有相应的应用。全英文授课的方式为我们以后阅读英文文献和听英文报告打下了一定的基础，但是如果有条件提供一些字幕，不管是英文还是中文，都会对同学们的学习有很大的帮助。又或者如果是线下上课，也可以相应推荐一些中文参考书帮助同学们过渡。

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### COMMENT 5

科学计算前沿选讲这门课程主要讲了一维对流扩散方程的数值算法和随机微分方程数值方法两个部分，由 Martin 教授和 Yanzhao Cao 教授分别通过线上进行主讲。曹婉容老师主持线上授课，同时也在 QQ 群里和大家进行沟通。老师们既专业又细致，认真授课，耐心解答学生疑问。作业通过群文件下发，过程清



晰，同时给大家讨论的环境，批改及时。通过本次学习，我了解了科学计算的其他前沿领域，同时对英文课程和阅读英文文献有了更强的适应能力。

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#### COMMENT 6

虽然因为各种原因课程是线上进行的，但是总的来说，讲课的两位教授都非常地认真负责，不仅讲方程的数值计算讲得清楚有条理，还提供了自己的讲义供我们复习参考，耐心地解答同学们的问题。当然，如果有更多的课堂互动的方式或者可以线下授课，并且讲课的内容在基础知识之余可以涉及相关学科发展前沿信息的话，这门课会有更大的吸引力。

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#### COMMENT 7

非常感谢 Martin 教授和曹延昭老师，两位老师的授课十分认真，暑期课程都是线上形式，但是曹婉容老师以及各位助教通过建立 qq 群，使得同学们不懂的问题也得到了及时的解决。Martin 教授讲解的一维对流扩散方程的数值算法，关于教学重点的指导也十分明确，让我了解了更多前沿的知识，整个过程中授课内容十分的丰富，全程英文授课，也提高了我的英文能力，通过布置课后作业，强调课上重点内容，也提高了我们解决困难的能力。曹老师从基础讲起，感觉就和平时我们上课一样，每一节课都会解答大家课上、课后、以及作业的问题，十分有耐心。在这个过程中，我夯实了基础，也意识到自己还有很多知识要继续深入学习。

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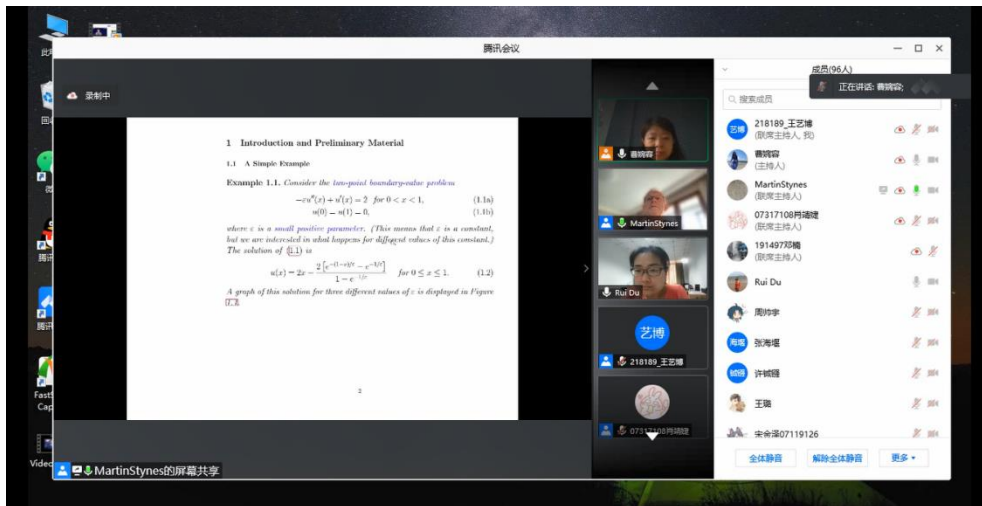
#### COMMENT 8

四周的暑期学校课程，我们简单了解了随机微分方程，学会了用 matlab 模拟出布朗运动的轨道，并学习了多种计算微分方程的方法，收获颇多。科学计算这门课程由多个外教进行全英文的授课，在外教们放慢速度之后，我是能够跟上课堂节奏的，但可能还是会因为对很多数学名词的英文表达较为生疏导致没跟上课堂的节奏。此外，还是希望之后的暑期课堂能提供课堂的录屏，一来是能帮助课堂上没有跟上的同学学习和巩固知识，二来也能让同学在课程结束过后能回顾关于微分方程的知识。

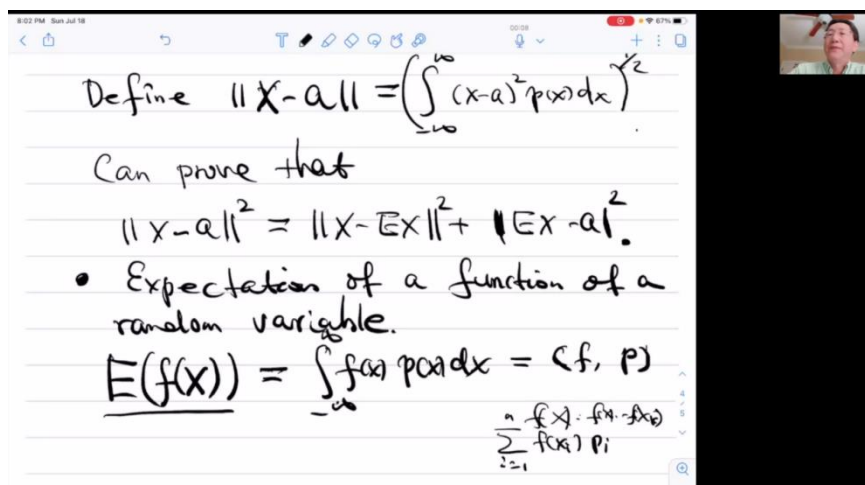
## FEEDBACK FROM TEACHERS

### 1) 课程特色与优点

a) 两位国际著名学者领衔授课，深入浅出讲授科学计算前沿领域热点问题。



Martin Stynes 教授讲授的“一维对流扩散问题的有限差分方法”围绕着 Stynes 教授著作中的前四章，从问题的背景出发，通过生动的算例让同学们们了解了奇异摄动对于数值求解的影响，再对于问题本身进行严谨的理论分析，估计解本身、解的导数在边界点处的性态分析数值求解该问题的困难和产生边界层的原因，最后引出求解该问题的有限差分方法以及有效的 Shishkin 型步长。Stynes 教授对于课程进行了非常精心的准备，整体课程设计完整，对于课程内容和节奏的把握恰到好处，讲解详略得当，非常清晰。并且每次课后都精心布置作业，帮助同学们巩固所学内容，应用所学的内容挑战具有有一点小难度的问题，并对课内知识进行延展和补充。



Yanzhao Cao 教授讲授的“随机微分方程数值方法”从测度论的角度介绍随机变量的定义为起点，介绍了条件期望、布朗运动的定义以及随机微积分的基础知识，引入随机微分方程解的存在唯一性和正则性，在此基础上介绍了求解随机微分方程的数值方法。Yanzhao Cao 教授采用手写板的方式边讲边写，特别强调思考方法和处理问题的方法，对于同一定义从不同角度解读，“授之以渔”，在讲授布朗运动时现场演示二维布朗运动的图像，并设计算例揭示随机扰动能够对一些系统起到稳定的作用，使人印象深刻。

b) 课程管理到位，多元化评价学生的学习成绩，及时反馈，保障学生取得良好的学习效果。

该课程的两个部分分别安排三名助教全程听课，协助同学向教授提问、在 QQ 群中随时回答同学们提出的疑问，分三组批改作业，及时对于作业做出反馈。负责课程组织的曹婉容老师在 Stynes 教授全英文授课期间，通过课程小结的方式梳理教学内容脉络，讲解重难点和答疑，帮助同学们快速适应在全英文环境下学习一门新课程。





## 2) 不足之处

组织教学过程中发现有部分同学的确对于科学计算类课程不感兴趣，因此学习热情欠缺，不能自觉预习和复习，在全英文教学环境下学习困难增加。如果能够允许学生自由选课，让每位同学能够根据自己的学习兴趣和需要自由选课，暑期学校将会发挥更好的作用。

## COURSE 3: CATEGORICAL DATA ANALYSIS

### Hours/Credits

24 hours (July 1—August 1, 2021)/ 1 credit

Wednesday and Friday 8:00 am – 10:35 am

### Description

The content mainly includes semi-parametric and non-parametric statistics, robust statistical models, high-latitude data and statistical analysis of big data, etc. Semi-parametric and non-parametric statistical models have a wide range of applications, and their assumptions are weaker than traditional parametric models, so they are more widely used especially in the era of big data, when statistical inferences tend to be more accurate. The data collected today often have outliers, and traditional statistical inferences such as the least square method for these outliers are very unstable and often lead to false inferences. Robust statistical models are not affected by these outliers and can provide robust and reliable statistical inferences. In the era of big data, a lot of data is high latitude. Traditional statistical analysis methods are often not applicable at this time. This course will introduce a series of high latitude statistical methods and some big data statistical calculation methods.

### Instructor



主讲人：姚卫鑫

Weixin Yao(University of California, Riverside)

<https://faculty.ucr.edu/~weixiny/index.html>

美国加州大学河滨分校统计系教授，主要研究混合模型、非参数和半参数模型、纵向数据分析、稳健估计、高维度建模、变量的选择和降维，发表论文 200 篇，是多个学术杂志的编辑。

### PREREQUISITES

Calculus, Linear Algebra, Differential Equations, Real Analysis, Complex Analysis, Probability Theory, Mathematical Statistics, Random Processes

## COURSE OBJECTIVES

This course will introduce a series of high latitude statistical methods and some big data statistical calculation methods.

## CLASS SCHEDULE

|             |   |
|-------------|---|
| Hours 1-2   | Introduction to Bayes rules, parameter estimation, and model selection, including some motivating examples. |
| Hours 3-4   | Uncertainty propagation and classic sampling method   |
| Hours 5-6   | Classic matching method and Bayesian learning   |
| Hours 7-8   | Expressing a priori uncertainty: general principles, reduced dimensions                                     |
| Hours 9-10  | Numerical methods for anti-problems and data assimilation   |
| Hours 11-12 | Design optimization under uncertainty   |
| Hours 13-14 | Set-up and objectives   |
| Hours 15-16 | Decision-theoretic framework for classification   |
| Hours 17-18 | Data and ambitions  |
| Hours 19-20 | Learning from data  |
| Hours 21-22 | Discriminant Analysis   |
| Hours 23-24 | Logistic Regression   |
| Hours 25-26 | Resampling methods  |
| Hours 27-28 | The validation set approach   |
| Hours 29-30 | Linear Regression   |
| Hours 31-32 | Model/Variable Selection  |
| Hours 33-34 | Robust Linear Regression  |
| Hours 35    | Nonparametric/semiparametric Regression   |

## FEEDBACK FROM STUDENTS

### COMMENT 1

姚老师讲解得很细致生动，通过这门课，我对于现代统计学的知识有了更直观深入的了解。每周两次课，每次三个课时的安排也比较合理。

但由于上这门课的学生为数学学院大二的三个专业的所有学生，信息与计算科学以及应数专业的学生没有上过随机过程等统计的专业课，知识储备量不够，且对于绝大多数学生来说，这是第一次上全英文的课程，本身就是非常大的挑战，还是在同时上三门全英文的课程外加一门编程算法类的课程的情况下。同时，刚刚结束春季长学期，同学们的身心还没有很好的调整，又由于疫情影响，只能通过 zoom 进行线上授课，这对于学生，老师及课程本身都带来了一定的困扰。

我认为属性数据分析这门课本身是比较有趣并且作为暑期学校的课程考核难度也比较合理的,但是可能更加适合于作为统计知识储备更为丰富的统计学专业的同学的必修课,其他两个专业的有能力和兴趣的同学可以进行选修或旁听。并且如果能放在暑期后秋季长学期之前,进行线下授课效果肯定会更好。

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#### COMMENT 2

整体来说不错,氛围比较轻松,也不会有太多压力,还能了解到相关知识。建议的话就是:相关知识更多的是介绍,没有很深入,只是了解,希望能挑几个重点讲解一下。

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#### COMMENT 3

四周的属性数据分析暑期学校课程,我们学习到了更多概率、统计的知识,是对我们刚学习完的概率论、数理统计课程的补充与延伸。全英文授课对我们是好事,不过在课程之初有些困难,难以理解授课内容,难以准确看懂讲义,适应一段时间后有所改善,能听懂的内容会变多。在大家适应全英文的艰难磨合期,姚教授也会用中文为我们讲解一些难懂的知识,相比之下还是中文更容易接受。此外,还是希望之后的线上教学能提供课堂的录屏,这样更能帮助同学学习和巩固知识。

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#### COMMENT 4

本课程主要偏向于科普类型,内容还是比较易于理解的,没有给学生太大的压力。老师有目标意识,知道学生学的是什么、以及怎样指导学生学习,总体上还是不错的。

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#### COMMENT 5

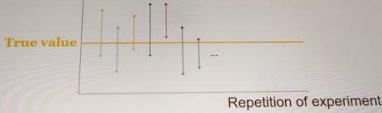
老师语速适中,内容讲解详细,经常用实例帮助同学理解重难点。考核的频次与难度适中,同学们反响较好。英文讲课阶段同学们不太适应,跟学有难度,但英文讲课中文举例的形式对同学们的课程理解有很大帮助。

### FEEDBACK FROM TEACHERS

《属性数据分析》这门课是由加州大学河滨分校姚卫鑫教授讲授,全校本科生和研究生共 90 多人参与了这门课的学习。姚卫鑫教授用英文主要讲解了数据的收集、统计推断、线性回归分析和多重线性回归分析等前沿内容,在讲解的过

程中都穿插了很多例子，及时停下来回答学生提出的问题，学生通过这门课的学习一方面提前感受全英文授课，另一方面在学知识的过程中开阔了视野。

### Confidence Interval



• Confidence interval is an interval that includes true parameter value  $(1 - \alpha)\%$  of the times over repeated sampling.

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### Feature Screening for Ultrahigh Dimensional Data

- Goal: reduce dimensionality of predictors from a large or huge scale to a relatively large scale by a fast and efficient screening method.
- Method: use some marginal utility measures, such as correlation, to screen predictors.
- Fan and Lv (2008) proposed Sure Independence Screening (SIS) procedure for linear model.

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### Estimation Methods

- Model:  $y_i = m(u_i) + x_i^T \beta + \epsilon_i$
- In a matrix format:  $y = \mathbf{m} + \mathbf{X}\beta + \epsilon$ ,
  - ◇  $y = (y_1, \dots, y_n)^T$ ,  $\epsilon = (\epsilon_1, \dots, \epsilon_n)^T$ ,  $\mathbf{m} = (m(u_1), \dots, m(u_n))^T$
  - ◇  $\mathbf{X} = (x_1, \dots, x_n)^T$
- Profile least squares method
  - ◇ Given  $\beta$ , the local linear estimate of  $m$  can be expressed as  $\hat{m} = S_n(\mathbf{X})(y - \mathbf{X}\beta)$
  - ◇  $\hat{m} = S_n(\mathbf{X})(y - \mathbf{X}\beta) + \mathbf{X}\beta + \epsilon$
  - ◇  $(I - S_n(\mathbf{X}))y = (I - S_n(\mathbf{X}))\mathbf{X}\beta + \epsilon$
  - ◇ Find the LSE of  $\beta$ , denoted by  $\hat{\beta}$ .
  - ◇  $\hat{m} = S_n(\mathbf{X})(y - \mathbf{X}\hat{\beta})$

*Linear smoother*

$$y = \mathbf{X}\beta + \epsilon$$

$$\hat{\beta} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T y$$

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### 4. Rejection Region:

**NOTE:**  $g = \#$  of parameters estimated from the data.

Since the data was NOT used to obtain the proportions (expected frequencies),  $g = 0$

Let  $\alpha = 0.05$ ,

$df = k - g - 1 = 4 - 0 - 1 = 3$

Reject  $H_0$  if

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