东南大学 2021 年国际暑期学校项目介绍

Introduction of SEU International Summer School Program

项目主题（Theme)

东南大学 2021 年数学与统计国际暑期课程
2021 International Summer Courses on Mathematics and Statistics, Southeast University

项目概述(Overview)

本课程将从数学与统计两个学科前沿方向选取适当的模型案例作为载体，介绍相关领域的最新研究成果，提升学生对知识的理解，强化学生对知识的运用。本课程的特色是理论与应用并重，并体现学科交叉融合。项目包含三门 24 学时的短课程，每门课程 1 学分，采用线上授课方式。

This program will select appropriate problem models from the cutting-edge aspects of mathematics and statistics respectively, and introduce the latest research results in the related fields, in order to improve the understanding and utilization of knowledge for students. The emphasis of both theory and application is the highlight of this course. In addition, the reflection of the interdisciplinary cross-integration is also the main goal of this course. The program consists of three 24-period short online courses with 1 credit for each course.

日程安排（暂定）(Schedule (Draft))

2021 年 7 月 5 日— 2021 年 8 月 1 日
July 5, 2021 — August 1, 2021

1 Mini Course：Selected Topics in Modern Mathematics

- **Hours/Credits**: 24 hours/ 1 credit

- **Lecturer:**

We will invite the teaching team from University of Luxembourg to teach this course, including:

Geometry：Prof. Dr. Jean-Marc Schlenker;

Analysis： Dr. Fei Pu;
Algebra： Prof. Dr. Antonella Perucca (course 1) and Prof. Dr. Gabor Wiese (course 2);
Probability and Statistics： Prof. Dr. Ivan Nourdin (course 1) and Prof. Dr. Mark Podolskij (course 2).

主讲教师

本课程将邀请由卢森堡大学理学院院长 Jean-Marc Schlenker 教授领衔的数学系教学团队前来授课。

- **Description**

======== GEOMETRY ========

Teacher: Prof. Dr. Jean-Marc Schlenker

Title: The geometry of polyhedra in Euclidean space

Abstract: We intend to present some classical and more recent results on the geometry of convex polyhedra in Euclidean space, as well as some open problems of current interest. The course could fit over 2 sessions of 135mn. Assessment could be done through a few multiple-choice of numerical-answer questions in a moodle-type test.

======== ANALYSIS ========

Teacher: Dr. Fei Pu

Title: Basics of Fourier Analysis

Abstract: I am planning to follow Stein's book to present some basic materials on Fourier Analysis and the key words are: Fourier inversion, Plancherel identity, Poisson summation formula, Theta and zeta functions.

======== ALGEBRA ========

Teachers: Prof. Dr. Antonella Perucca (course 1) and Prof. Dr. Gabor Wiese (course 2)

Title: Finite fields: from the cyclicity of the unit group to Artin's conjecture on primitive roots, Gauss' quadratic reciprocity law, primality tests and the Langlands program

Abstract:
Part 1 - Artin’s Conjecture for primitive roots and related problems (A. Perucca)
We start by considering the unit group (Z/pZ)* of the integers modulo a prime number p, and then

investigate the multiplicative order and index of an element in this group. By varying the prime number, distribution questions naturally arise, the most famous being the one addressed in Artin's Conjecture for primitive roots. To understand the conjecture and its heuristics we introduce cyclotomic number fields and Kummer extensions. To conclude we present recent results on this topic obtained by mathematicians in Luxembourg.

Part 2 - A primality test, quadratic reciprocity, and more general reciprocity laws (G. Wiese)
From Part 1, we know that half of the elements in (Z/pZ)* are squares and half are non-squares. The famous quadratic reciprocity law conjectured by Euler and proved by Gauss relates this for two primes: say p_1,p_2 are two primes that are 1 mod 4; then p_1 is a square mod p_2 if and only if p_2 is a square mod p_1. This law can be proved using cyclotomic fields, introduced in Part 1. As a practical application of quadratic reciprocity, we introduce the Solovay-Strassen primality test for deciding if a given positive integer is a prime number or not.
In a final part, we give some hints on generalisations of quadratic reciprocity leading us (vaguely) to the Langlands program.

======== PROBABILITY AND STATISTICS ========

Teachers: Prof. Dr. Ivan Nourdin (course 1) and Prof. Dr. Mark Podolskij (course 2)

Title: Large random matrices

Abstract:
Part 1: "Free probability and large random matrices"
- Reminder of the classical central limit theorem
- Random matrices
- Concept of classical and free independence
- Stieltjes transform
- Semicircular and Marcenko-Pastur laws
- Voiculescu's theorem, and an alternative proof of Wigner's theorem
- Classic and free Brownian motion

Part 2: "Estimation of large covariance matrices"
-Empirical covariance matrices
-Principal component analysis
-Asymptotic theory for empirical eigenvalues
-Estimation of high-dimensional covariance matrices
-Relations to random matrix theory

课程描述

"现代数学选讲"是为数学各专业开设的一门必修课程。课程内容将由主讲

教师根据自身专业特长，结合现代数学的发展，遴选分析、代数、几何等分支中的经典热门话题，围绕几个数学问题为学生详细介绍其研究历史、处理手法和最新进展，开阔学生的学术视野，提升学生对于数学的学习兴趣。

- **Tentative schedule**

Day	Start time	End time	Teacher	Topic
July 5 (Monday)	8:45am (UL time) 2:45pm (SEU time)	11am (UL time) 5pm (SEU time)	Prof. Dr. Jean-Marc Schlenker	Geometry
July 8 (Thursday)	8:45am (UL time) 2:45pm (SEU time)	11am (UL time) 5pm (SEU time)	Prof. Dr. Jean-Marc Schlenker	Geometry
July 12 (Monday)	8:45am (UL time) 2:45pm (SEU time)	11am (UL time) 5pm (SEU time)	Prof. Dr. Antonella Perucca	Algebra
July 15 (Thursday)	8:45am (UL time) 2:45pm (SEU time)	11am (UL time) 5pm (SEU time)	Prof. Dr. Gabor Wiese	Algebra
July 19 (Monday)	8:45am (UL time) 2:45pm (SEU time)	11am (UL time) 5pm (SEU time)	Dr Fei Pu	Analysis
July 22 (Thursday)	8:45am (UL time) 2:45pm (SEU time)	11am (UL time) 5pm (SEU time)	Dr Fei Pu	Analysis
July 26 (Monday)	8:45am (UL time) 2:45pm (SEU time)	11am (UL time) 5pm (SEU time)	Prof. Dr. Ivan Nourdin	Probability and Statistics
July 29 (Thursday)	8:45am (UL time) 2:45pm (SEU time)	11am (UL time) 5pm (SEU time)	Prof. Dr. Mark Podolskij	Probability and Statistics

2 Mini Course: Selected Topics in Frontier of Scientific Computation

2.1 Mini Course: Selected Topics in Frontier of Scientific Computation (Part I) Topic: Machine Learning and Design optimization under uncertainty

- **Hours/Credits:** 12 hours/ 0.5 credit

- **Lecturer:**

Matin Stynes

Beijing Computational Science Research Center
(m.stynes@csrc.ac.cn，http://www.csrc.ac.cn/en/people/faculty/151.html）

主讲教师：Martine Stynes，北京计算科学研究中心国家“千人计划”专家，奇异摄动微分方程数值方法领域国际领军学者，曾任工业和应用数学学会（SIAM）英国和爱尔兰分会主席（2003-2005），曾任 SIAM Journal on Numerical Analysis 编委，目前担任 Advances in Computational Mathematics，Computational Methods in Applied Mathematics，Mathematical Proceedings of the Royal Irish Academy 等学术期刊编委。
邮箱 m.stynes@csrc.ac.cn，主页 http://www.csrc.ac.cn/en/people/faculty/151.html

- **Description**

In convection-diffusion problems, when the elliptic operators are multiplied by some parameter that is allowed to be close to zero, the first-order convective derivatives no longer play a relatively minor role is the system. Instead, it also has a strong influence on the solution of the boundary value problem. The convection-diffusion problems have been widely used in application, such as the Black-Scholes equation in finance, the linearized Navier-Stokes equation with large Reynolds number. The goal of this course is to introduce the background of the one-dimensional convection-diffusion equations and the basic theory of analytical solution and the finite difference method.

课程描述

在对流-扩散问题中，当椭圆算子前面具有一个趋近于零的系数，对流过程不再处于次要地位，而是在系统中占优，对于边值问题的解产生强大的影响。此类问题有很广泛的应用背景，例如金融中的 Black-Scholes 方程，具有大雷诺数的 Navier-Stokes 方程线性化后也会转化成此类问题。本课程介绍一维对流-扩散问题的背景知识、基本理论和有限差分方法。

- **Prerequisites**

Calculus, Linear Algebra, Differential Equations, Numerical Analysis. Students are strongly encouraged to use MATLAB for programming.

- **Textbooks**

Martin Stynes, David Stynes, Convection-Diffusion Problems：An Introduction to Their Analysis and Numerical Solution, The American Mathematical Society.

- **Course objectives**

After this course, students should be able to

- Understand the background of the convection-diffusion problems
- Understand the fundamental theory of the one-dimensional convection- diffusion problems
- Master the finite difference method for the one-dimensional convection- diffusion problems and its convergence analysis

- **Class schedule**

Hours 1-2	Introduction to the convection-diffusion problems by some motivating examples
Hours 3-4	Maximum principle and asymptotic expansion
Hours 5-6	Asymptotic analysis to the convection-diffusion problems, Green's formula
Hours 7-8	A priori bounds on the solution and decompositions of the solution
Hours 9-10	Upwinding scheme for solving the convection-diffusion problems
Hours 11-12	Shishkin meshes, uniformly convergent schemes

- **Evaluation methods：**

Project.

2.2 Mini Course: Selected Topics in Frontier of Scientific Computation (Part II)

Topic: Introduction to Numerical Methods for Stochastic Differential Equations

- **Hours/Credits:** 12 hours/ 0.5 credit

- **Lecturer:**

Yanzhao Cao
Department of Mathematics & Statistics,
Auburn University
(yzc0009@auburn.edu, http://webhome.auburn.edu/~yzc0009/)

主讲教师：曹延昭， 美国奥本大学数学与统计系教授，国家天元数学东北中心执行委员会主任。1983 年毕业于吉林大学数学系，1996 年获弗吉尼亚理工学院数学博士学位，主要从事偏微分方程和积分方程数值解法、随机偏微分方程数值解、非线性滤波、不确定性量化等领域的研究，部分重要研究成果发表在《SIAM J. Numer. Anal.》、《Numer. Math.》、《Math. Comp.》、《IMA J. Numer. Anal.》等计算数学国际顶尖杂志。现担任包括计算数学国际顶尖期刊《SIAM J. Numer. Anal.》在内的多个学术期刊编委。
(邮箱 yzc0009@auburn.edu, 主页 http://webhome.auburn.edu/~yzc0009/)

- **Description**

In this short course, I will introduce numerical methods for stochastic differential equations, which have been used widely used in biology, finance and engineering. Topics include Brownian motion and stochastic calculus in linear and nonlinear equations, analytic and numerical methods for SDEs, and parameter estimation for SDEs.

课程描述

随机微分方程在生物、金融、工程中有着广泛的应用，本课程介绍求解随机微分方程的数值方法，包括布朗运动、线性和非线性方程的随机计算、随机微分方程解析/数值解、随机微分方程的参数估计。

- **Prerequisites**

Calculus, linear algebra, differential equations and probability. Students are strongly encouraged to use MATLAB for programming.

- **Textbooks**

There will be no textbooks but lecture notes will be provided.

- **Course objectives**

After this course, students should be able to

- Learn the background and application to the mathematical models with random parameters or stochastic disturbance
- Master basic algorithms for solving problems with stochastic disturbance or random parameters
- Learn the algorithms to stochastic computation based on machine learning

- **Class schedule**

Hours 1-2	Introduction to stochastic differential equations, including some motivating examples.
Hours 3-4	Random walk, Brownian motion and stochastic calculus, and stochastic differential equations
Hours 5-6	Strong solutions, Well-posedness, Solution techniques
Hours 7-8	Basic concepts of numerical methods for stochastic differential equations, simulation of white and color noises Numerical methods for linear equations: stability and convergence
Hours 9-10	Numerical methods for nonlinear equations. Stiffness and treatment
Hours 11-12	Parameter estimation or stochastic differential equations

- **Evaluation methods：**

Project.

3 Mini Course: Categorical Data Analysis

- **Hours/Credits:** 24 hours/ 1 credit

- **Lecturer:**

Prof. Weixin Yao
Department of Statistics, University of California, Riverside

主讲教师：姚卫鑫，美国加州大学河滨分校统计系教授，主要研究混合模型、非参数和半参数模型、纵向数据分析、稳健估计、高维度建模、变量的选择和降维，发表论文 200 篇，是多个杂志的编辑。

- **Description:**

The content mainly includes semi-parametric and non-parametric statistics, robust statistical models, high-latitude data and statistical analysis of big data, etc. Semi-parametric and non-parametric statistical models have a wide range of applications, and their assumptions are weaker than traditional parametric models, so they are more widely used especially in the era of big data, when statistical inferences tend to be more accurate. The data collected today often have outliers, and traditional statistical inferences such as the least square method for these outliers are very unstable and often lead to false inferences. Robust statistical models are not affected by these outliers and can provide robust and reliable statistical inferences. In the era of big data, a lot of data is high latitude. Traditional statistical analysis methods are often not applicable at this time. This course will introduce a series of high latitude statistical methods and some big data statistical calculation methods.

课程描述

内容主要包括半参数和非参数统计，稳健统计模型，高维数据和大数据的统计分析等。半参数和非参数统计模型有着广泛的应用，她们的假设条件比传统的参数模型更弱所以应用更广特别是在大数据时代，统计推断往往也更准确。现如今收集到的数据，往往有些异常点，传统的统计推断譬如最小二乘方法针对这些有异常点的数据会非常不稳定，往往会得出错误的推断。稳健统计模型不受这些异常点的影响，并能够提供稳健可靠的统计推断。在大数据时代，很多数据都是高维的。传统的统计分析方法在这个时候往往不适用。这个课程会介绍一系列高维统计方法和一些大数据统计计算的方法。

- **Prerequisites:**

Calculus, Linear Algebra, Differential Equations, Real Analysis, Complex Analysis, Probability Theory, Mathematical Statistics, Random Processes

- **Textbooks:** Notes

- **Schedule:** July 5, 2021 — August 1, 2021

- **Evaluation:** Project or Paper Test

计划招生人数(Number of Participants)

200

申请要求(Application Requirements)

主要面向数学类专业和统计学专业修完二年级课程的本科生
It is mainly for the undergraduate students who have finished the second year courses in mathematics or statistics

申请截止时间(Application Deadline)

2021 年 6 月 15 日

June 15, 2021

主办/承办单位(Host & Organizer)

东南大学主办 / 数学学院承办
Southeast University / School of Mathematics

联系人及联系方式(Contact Information)

魏婉梦 老师 / wwm@seu.edu.cn
Miss Wanmeng Wei / wwm@seu.edu.cn